Algebra Essentials

College Algebra

Mathematics, the Language of Science

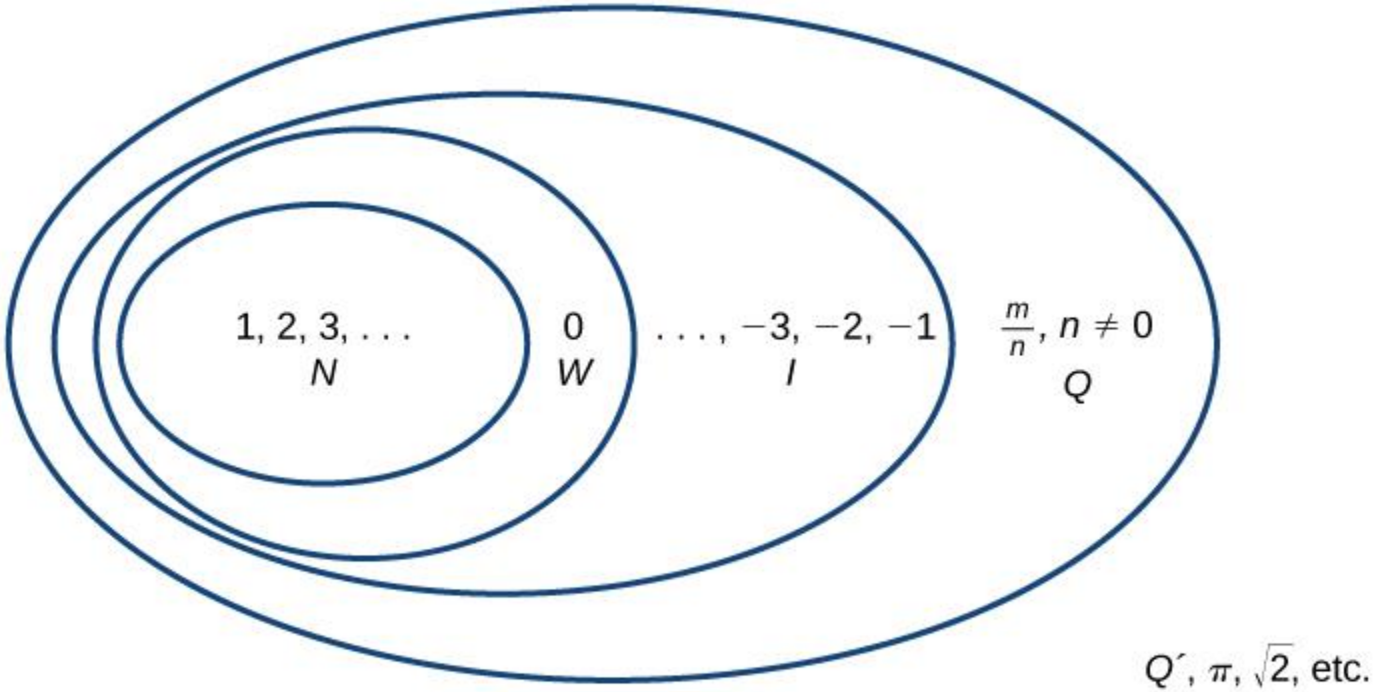
- Earliest use of numbers occurred 100 centuries ago in the Middle East to count items
- Farmers, cattlemen, and tradesmen used tokens, stones, or markers to signify a single item
- Made commerce possible and lead to improved communications and spread of civilization

Yuan Dynasty iron magic square showing Persian/ Arabic numbers (ca. 1271 – 1368)

Real Numbers

- **Natural numbers:** numbers used for counting: $\{1,2,3,\ldots\}$
- **Whole numbers:** natural numbers plus zero: $\{0,1,2,3,\ldots\}$
- **Integers:** adds negative natural numbers to the set of whole numbers: $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$
- **Rational numbers** includes fractions written as $\{\frac{m}{n} \mid m$ and n are integers and $n \neq 0\}$
- **Irrational numbers** is the set of numbers that are not rational, are nonrepeating, and are nonterminating: $\{h \mid h$ is not a rational number$\}$

Real Numbers



There is a subset relationship between the sets of numbers

N : the set of natural numbers, *W* : the set of whole numbers, *I* : the set of integers, *Q* : the set of rational numbers, *Q* ´: the set of irrational numbers

Order of Operations

Operations in mathematical expressions must be evaluated in a systematic order, which can be simplified using the acronym **PEMDAS**:

- **P**(arentheses)
- **E**(xponents)
- **M**(ultiplication) and **D**(ivision)
- **A**(ddition) and **S**(ubtraction)

Simplify a Mathematical Expression using the Order of Operations

1. Simplify any expressions within grouping symbols

2. Simplify any expressions containing exponents or radicals

3. Perform any multiplication and division in order, from left to right

4. Perform any addition and subtraction in order, from left to right

Properties of Real Numbers

	Addition	**Multiplication**
Commutative Property	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative Property	$a + (b + c) = (a + b) + c$	$a\,(bc) = (ab)\,c$
Distributive Property	$a \cdot (b + c) = a \cdot b + a \cdot c$	
Identity Property	There exists a unique real number called the additive identity, 0, such that, for any real number a $$a + 0 = a$$	There exists a unique real number called the multiplicative identity, 1, such that, for any real number a $$a \cdot 1 = a$$
Inverse Property	Every real number a has an additive inverse, or opposite, denoted $-a$, such that $$a + (-a) = 0$$	Every nonzero real number a has a multiplicative inverse, or reciprocal, denoted $\frac{1}{a}$, such that $$a \cdot \left(\frac{1}{a}\right) = 1$$

The properties hold for real numbers *a*, *b*, and *c*

Evaluate and Simplify Algebraic Expressions

- **Algebraic expression:** collection of constants and variables joined by the algebraic operations of addition, subtraction, multiplication, and division
- **Equation:** mathematical statement indicating that two expressions are equal
- **Formula:** equation expressing a relationship between constant and variable quantities

Simplify algebraic expression to make it easier to evaluate using the properties of real numbers. We can use the same properties in formulas because they contain algebraic expressions.

Desmos Interactive

Topic: explore how changing length or width changes the area of a rectangle

https://www.desmos.com/calculator/eq1cow0lcj

Rule of Exponents

The Product Rule: For any real number a and natural numbers m and n, the product rule of exponents states that

$$a^m \cdot a^n = a^{m+n}$$

Rule of Exponents

The Quotient Rule: For any real number a and natural numbers $m > n$ the quotient rule of exponents states that

$$\frac{a^m}{a^n} = a^{m-n}$$

Rule of Exponents

The Power Rule: For any real number a and positive integers m and n, the power rule of exponents states that

$$(a^m)^n = a^{m \cdot n}$$

Zero and Negative Rule of Exponents

Zero Exponent Rule: For any nonzero real number a, the zero exponent rule of exponents states that

$$a^0 = 1$$

Negative Exponent Rule: For any nonzero real number a and natural number n, the negative rule of exponents states that

$$a^{-n} = \frac{1}{a^n}$$

The Power of a Product Rule of Exponents

For any real numbers a and b and any integer n, the power of a product rule of exponents states that

$$(ab)^n = a^n b^n$$

The Power of a Quotient Rule of Exponents

For any real numbers a and b and any integer n, the power of a product rule of exponents states that

$$(\frac{a}{b})^n = \frac{a^n}{b^n}$$

Scientific Notation

- A number is written in **scientific notation** if it is written in the form $a \times 10^n$, where $1 \leq |a| < 10$ and n is an integer
- To convert a number in **scientific notation** to standard notation, simply reverse the process
- Scientific notation, used with the rules of exponents, makes calculating with large or small numbers much easier than doing so using standard notation

Summary: Exponents and Scientific Notation

Rule of Exponents For nonzero real numbers a and b and integers m and n	
Product Rule	$a^m \cdot a^n = a^{m+n}$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$
Power Rule	$(a^m)^n = a^{m \cdot n}$
Zero Exponent Rule	$a^0 = 1$
Negative Rule	$a^{-n} = \frac{1}{a^n}$
Power of a Product Rule	$(a \cdot b)^n = a^n \cdot b^n$
Power of a Quotient Rule	$(\frac{a}{b})^n = \frac{a^n}{b^n}$

Square Root

- The **principal square root** of a is the nonnegative number that, when multiplied by itself, equals a.
- It is written as a **radical expression**, with a symbol called a **radical** over the term called the **radicand**: $\sqrt{a}$

Product Rule for Simplifying Square Roots

If a and b are nonnegative, the square root of the product ab is equal to the product of the square roots of a and b

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

The Quotient Rule for Simplifying Square Roots

The square root of the quotient $\frac{a}{b}$ is equal to the quotient of the square roots of a and b, where $b \neq 0$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Operations on Square Roots

Radical expression requiring addition or subtraction of square roots:

1. Simplify each radical expression
2. Add or subtract expressions with equal radicands

Example: $5\sqrt{12} + 2\sqrt{3}$

- We can rewrite $5\sqrt{12}$ as $5\sqrt{4 \cdot 3}$
- Using the product rule, this becomes $5\sqrt{4}\sqrt{3}$, or $5 \cdot 2 \cdot \sqrt{3}$ or $10\sqrt{3}$
- Add the two terms that have the same radicand: $10\sqrt{3} + 2\sqrt{3} = 12\sqrt{3}$

Operations on Square Roots

Rationalize the denominator of an expression with a single square root radical term in the denominator

1. Multiply the numerator and denominator by the radical in the denominator
2. Simplify

Example: $\frac{2\sqrt{3}}{3\sqrt{10}}$

- Multiply both the numerator and denominator by $\sqrt{10}$
- Simplify: $\frac{2\sqrt{3}\cdot\sqrt{10}}{3\sqrt{10}\cdot\sqrt{10}} = \frac{2\sqrt{30}}{3\cdot 10} = \frac{\sqrt{30}}{15}$

Operations on Square Roots

Given an expression with a radical exponent, write the expression as a radical

1. Determine the power by looking at the numerator of the exponent
2. Determine the root by looking at the denominator of the exponent
3. Using the base as the radicand, raise the radicand to the power and use the root as the index

Example: $8^{\frac{5}{3}}$

- $(\sqrt[3]{8})^5 = 2^5 = 32$

Nth Roots and Rational Exponents

Nth Root: If a is a real number with at least one n th root, then the principal n th root of a, written as $\sqrt[n]{a}$, is the number with the same sign as a that, when raised to the n th power, equals a. The **index** of the radical is n.

Rational exponents: another way to express principal n th roots. The general form for converting between a radical expression with a radical symbol and one with a rational exponent is

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

Quick Review

- What is the difference between a natural number and a whole number?
- What is a rational number?
- What is exponential notation?
- What is the order of operations for performing calculations?
- What is the commutative property of addition?
- What is the associative property of multiplication?
- What is the inverse property of multiplication?
- What is an equation?
- What is the product rule for exponents?
- What is the power rule for exponents?
- What is scientific notation?
- What is a radical expression?
- How do you simplify the square root of a product